1. The random matrix model that provides the closest connection with Gauge theory is the Stieltjes-Wigert ensemble.

This is simple to prove. Consider the Hermitian random matrix ensemble:

$$P(x_1,...,x_N)=C_N\prod_{i<j}|x_i-x_j|^2\ \prod_{i=1}^{N}\mathrm{e}^{-V(x_i)}, \tag{1.1}$$

Instead of the very much studied Gaussian $V(x)=x^2$ or polynomial $V(x)=\sum_n a_n x^n$ cases, consider the log-normal case:

$$\omega(x)=\mathrm{e}^{-V(x)}=\frac{1}{\sqrt{\pi}}k\mathrm{e}^{-k^2\log^2 x}, \tag{1.2}$$

the ensemble is named Stieltjes-Wigert, due to the name of the associated orthogonal polynomials. Actually, this weight and its associated orthogonal polynomials have a distinguished story and many remarkable properties as well.

For the moment, let us just compute the partition function:

$$Z=\int\prod_{i=1}^{N}\frac{1}{\sqrt{\pi}}k\mathrm{e}^{-k^2\log^2 x_i}\prod_{i<j}|x_i-x_j|^2 \tag{1.3}$$

We do this in detail:

For this, we can use the following well-known result for the partition function in terms of the orthogonal polynomials :

$$Z=\int...\int\prod_{i=1}^{N}\omega(x_i)\,dx_i\prod_{i<l}(x_i-x_l)^2=\frac{N!}{\prod_{i=0}^{N-1}a_i^2}=N!a_0^{-2N}\prod_{i=1}^{N-1}\left(\left(\frac{a_{i-1}}{a_i}\right)^2\right)^{N-i}. \tag{1.4}$$

where the coefficient a_i is:

$$p_i(x)=a_i x^i+... \tag{1.5}$$

The first step is to compute the Z associated to the Stieltjes-Wigert orthogonal polynomials [**?**]. The coefficients are given by:

$$a_j=q^{(j+1/2)^2}\left\{(1-q)...\left(1-q^j\right)\right\}^{-1/2}, \tag{1.6}$$

then

$$\left(\frac{a_{j-1}}{a_j}\right)^2=q^{-4j}\left(1-q^j\right), \tag{1.7}$$

and $a_0=q^{1/4}$. Therefore:

$$\begin{aligned} Z_{sw} &= N!q^{-N/2}\prod_{j=1}^{N-1}q^{-4j(N-j)}\left(1-q^j\right)^{N-j} \\ &= N!q^{-\frac{1}{6}N(2N-1)(2N+1)}\prod_{j=1}^{N-1}\left(1-q^j\right)^{N-j}, \end{aligned} \tag{1.8}$$

Interestingly enough, it is known that the partition function of Chern-Simons theory on S^3 with $U(N)$ is essentially given by:

$$Z = \frac{\exp\left(-\frac{g_s}{12}N\left(N^2-1\right)\right)}{N!}\int\prod_{i<j}\left(2\sinh\frac{u_i-u_j}{2}\right)^2\prod_{i=1}^{N}\mathrm{e}^{-u_i^2/2g_s}\frac{du_i}{2\pi}, \tag{1.9}$$

Happily enough, these two models are simply related by:

$$u_i = \log x_i + N g_s,$$

Actually a mapping such as $u_i = \log x_i$ also works, due to rather nice properties of the Stieltjes-Wigert ensemble. Thus:

$$\begin{aligned} &\int\prod_{i=1}^{N}\frac{du_i}{2\pi}\exp\left(-\frac{u_i^2}{2g_s}\right)\prod_{i<j}\left(2\sinh\left(\frac{u_i-u_j}{2}\right)\right)^2 \qquad (1.10)\\ &= (2\pi)^{-N}\exp\left(\frac{-N^3 g_s}{2}\right)\int\prod_{i=1}^{N}dx_i\exp\left(-\frac{\log^2(x_i)}{2g_s}\right)\prod_{i<j}(x_i-x_j)^2. \end{aligned}$$

Then:

$$\mathcal{Z}\left(S^3\right) = \frac{\mathrm{e}^{-\frac{1}{12}g_s N\left(7N^2-1\right)}}{N!}\left(\frac{2\pi}{g_s}\right)^{-N/2}\mathcal{Z}_{sw} \tag{1.11}$$

Thus the partition function of the Hermitian Stieltjes-Wigert ensemble gives the partition function of a topological quantum field theory in three-dimensions. Namely, Chern-Simons theory on S^3 with gauge group $U(N)$. additionally, it seems that the Stieltjes-Wigert model posses many more remarkable properties in relationship with Chern-Simons theory. To make explicit contact with the typical expressions for the Chern-Simons partition functions we make some transformations on the product term in Z_{sw} :

$$\begin{aligned} \prod_{j=1}^{N-1}\left(1-q^j\right)^{N-j} &= \prod_{j=1}^{N-1}\left(2\sinh\frac{g_s j}{2}\right)^{N-j}\exp\left(-\frac{g_s j\,(N-j)}{2}\right)\\ (1.12)\qquad &= \exp\left(\frac{-1}{12}g_s N\left(N^2-1\right)\right)\exp\left(\frac{i\pi}{4}N(N-1)\right)\cdot\prod_{j=1}^{N-1}\left(2\sin\frac{g_s j}{2i}\right)^{N-j}. \end{aligned}$$

The final expression for the partition function is:

$$\begin{aligned} Z &= \frac{\exp\left(-\frac{1}{12}g_s N\left(N^2-1\right)\right)}{N!}Z_{\sinh} \qquad (1.13)\\ &= \left(\frac{g_s}{2\pi}\right)^{N/2}\exp\left(\frac{i\pi}{4}N(N-1)\right)\prod_{j=1}^{N-1}\left(2\sin\frac{g_s j}{2i}\right)^{N-j}, \end{aligned}$$

and since $g_s = \frac{2\pi i}{k+N}$, we finally find:

$$\mathbf{Z} = \mathbf{e}^{\frac{1}{4}i\pi N^2}(k+N)^{-N/2}\prod_{j=1}^{N-1}\left(2\sin\frac{\pi j}{k+N}\right)^{N-j}. \tag{1.14}$$